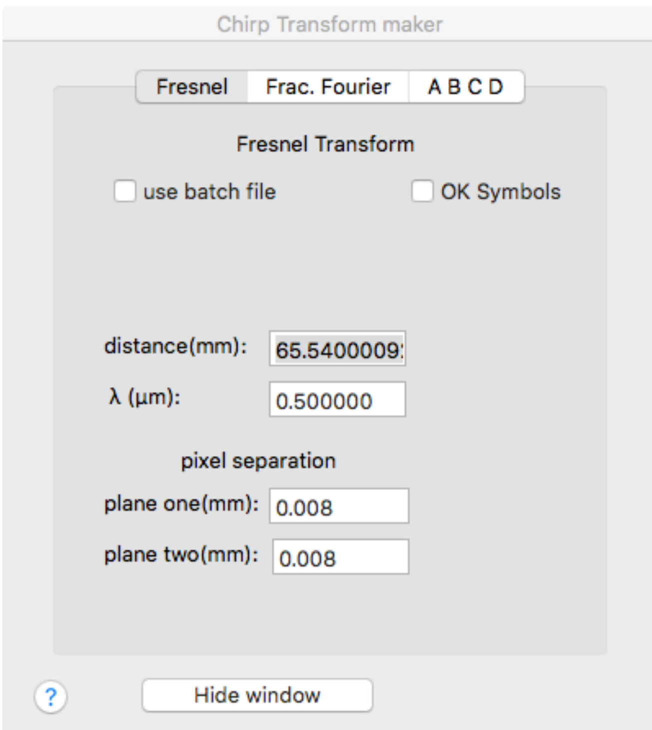
About Chirps

This Icon was made to deal with the need for speed in following the evolution of a light beam. It lacks the accuracy of the “Direct Propagation” icon which permits large magnification and which includes the the effects of a finite source pixel, but it is very much faster. When used on a Macintosh computer with several cores it will really fly.

The icon has three pages. All of the pages and their associated algorithms make use of multiple FFT’s and quadratic phase functions [aka lens functions or ZPF].

FRESNEL TRANSFORM



We think of the propagation through a distance “*d*” as represented by the ray matrix: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ this matrix represents an operation which in this instance is represented by two algorithms, one if the pixel separations are the different and another simpler one if the same.

if “Δ_1“ is the pixel center to center distance in plane one. “Δ_2“ is the equivalent for plane two. The wave length of the light is “λ”. The width (and height) of the array in pixels is “*N*”. The (*x,y*) pixel coordinates are (*n,m*). The calculation has five parts.

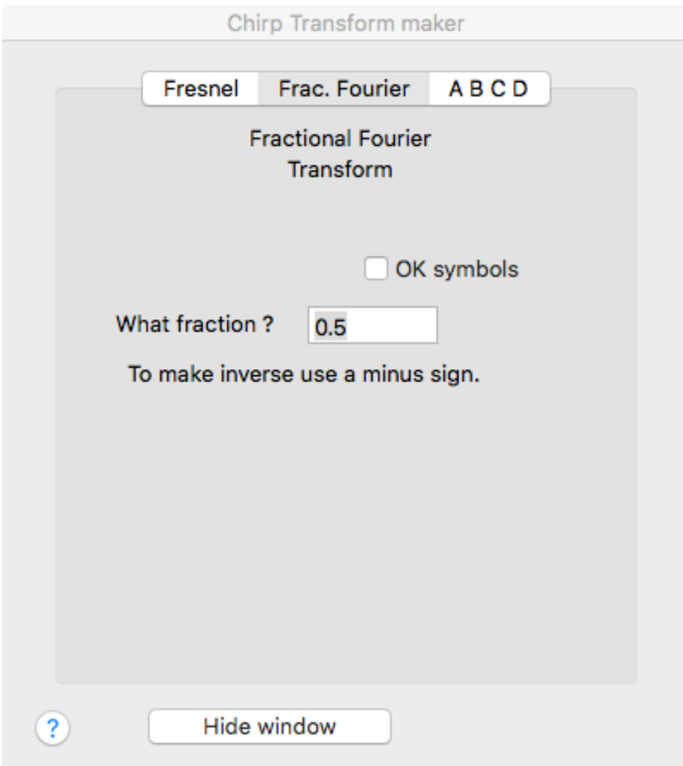
1) multiplication by a lens function.
2) a fast inverse Fourier transform.
3) another lens function.
4) a fast Fourier transform
5) a final lens function.

The lens functions have a phase defined as follows.

$$\phi_1 = \frac{\pi\Delta_1}{\lambda d}(\Delta_1 - \Delta_2)(n^2 + m^2)$$
$$\phi_3 = \frac{\pi\lambda d}{N^2\Delta_1\Delta_2}(n^2 + m^2)$$
$$\phi_5 = \frac{\pi\Delta_2}{\lambda d}(\Delta_2 - \Delta_1)(n^2 + m^2)$$

Steps one and five are skipped if Δ_1 equals Δ_2. You will find that you can magnify the output if Δ_2 is smaller than Δ_1 ,,just as in “direct propagation”. The batch file options works the same here as in the “direct propagation” icon. At each invocation of the icon a new record is read from a text file up to the next carriage return or newline. The first four values are entered into the text fields of the window. If the entry is zero then the original value in the text field is not replaced. This will continue until a zero is found in the first (distance) field. Execution of the script will then stop or the script will branch if a green alternate line originates at the icon.

FRACTIONAL FOURIER TRANSFORM



$\begin{pmatrix} \cos\theta & f_c \sin\theta \\ \frac{-\sin\theta}{f_c} & \cos\theta \end{pmatrix}$ is the ray matrix representing a Fractional Fourier Transform. A Fourier Transform corresponds to a choice of $\theta = \pi/2$. The number in this field is used to multiply $\pi/2$. This matrix is realized by the same five steps and the Fresnel Transform.

1)a lens function
2)a inverse FFT
3)a lens function
4)an FFT
5)a lens function

Using s as the fraction, phases for the lens functions are as follows.
In this case when the “Use oversize FFT” option is turned on, only the *N* in ϕ_3 changes, and it changes to 4*N*.

$$\phi_1 = \phi_5 = \frac{\pi(\cos(\pi s/2) - 1)}{N \sin(\pi s/2)}(n^2 + m^2)$$
$$\phi_3 = \frac{\pi \sin(\pi s/2)}{N}(n^2 + m^2)$$

$$\begin{pmatrix} 1 & 0 \\ \frac{\cos\alpha - 1}{f_c \sin\alpha} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{-1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-\sin\alpha}{f_c} & 1 \end{pmatrix} \begin{pmatrix} 0 & -f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\cos\alpha - 1}{f_c \sin\alpha} & 1 \end{pmatrix}$$

$$f_c = N\Delta^2/\lambda$$

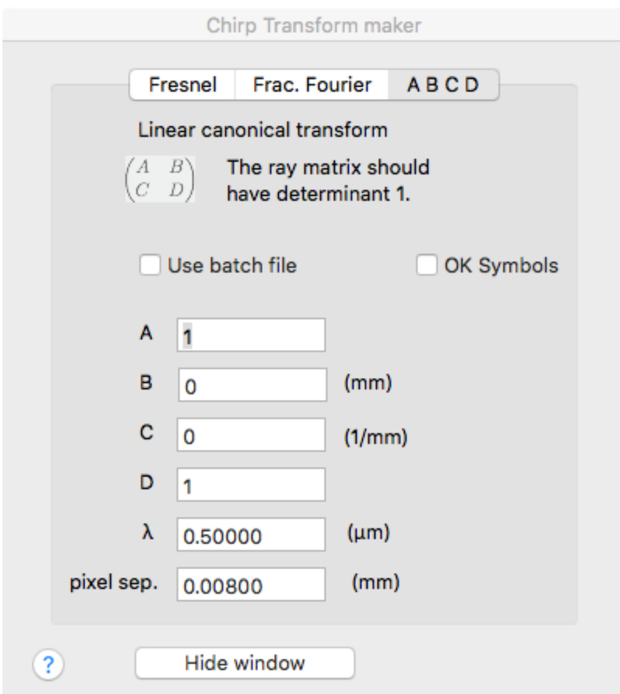
ABCD GENERAL LINEAR RAY MATRIX

This program uses two algorithms to calculate the general linear matrix. The matrix may be factored in may ways I have chosen two.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{D-1}{B} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-B}{f_c^2} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{A-1}{B} & 0 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0 & f_c \\ \frac{-1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{f_c-B}{f_c^2 D} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{-1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{D}{f_c} & 1 \end{pmatrix} \begin{pmatrix} 0 & -f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1+f_c C}{f_c D} & 1 \end{pmatrix}$$

The first of these is shorter but clearly fails if B = 0. Since the determinant is equal to 1 then B and D cannot both be zero. The value of B is the test that determines which algorithm is used. We use either 5 steps or 6 as the case may be.



Case B ≠ 0.
1) lens function
2) inverse FFT
3) lens function
4) FFT
5) lens function

$$\phi_1 = \frac{\pi\Delta^2(A-1)}{\lambda B}(n^2+m^2)$$

$$\phi_3 = \frac{-\pi B\lambda}{N^2\Delta^2}(n^2+m^2)$$

$$\phi_5 = \frac{\pi\Delta^2(D-1)}{\lambda B}(n^2+m^2)$$

Δ is the pixel separation & $f_c = N\Delta^2/\lambda$.

No attempt is made to assure that the determinant of the matrix is actually equal to one. You can cheat all you want as long as you get the right answer.

Case B = 0.
1) lens function
2) FFT inverse FFT
3) lens function
4) FFT
5) lens function.
6) FFT

$$\phi_1 = \frac{\pi}{ND}\left(\frac{N\Delta^2 C}{\lambda}+1\right)(n^2+m^2)$$

$$\phi_3 = \frac{\pi D}{N}(n^2+m^2)$$

$$\phi_5 = \frac{\pi}{ND}\left(1-\frac{B\lambda}{ND\Delta^2}\right)(n^2+m^2)$$

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The same pixel separation is used for both input and output planes.

The “Batch” option works like the batch option for “Fresnel”. At each invocation of the icon a new record is read from a text file up to the next carriage return or newline. It fills the fields from the top down on this page from that record. If the entries for “λ” or “Δ” are zero they are not changed from the previous value.

Illegal numbers will cause a branch not a crash (I hope).