

## About Chirps



### ABCD GENERAL LINEAR RAY MATRIX

This program uses two algorithms to calculate the general linear matrix. The matrix may be factored in many ways I have chosen two.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{D-1}{B} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-B}{f_c^2} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{A-1}{B} & 0 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0 & f_c \\ \frac{-1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{f_c-B}{f_c^2 D} & 1 \end{pmatrix} \begin{pmatrix} 0 & f_c \\ \frac{-1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{D}{f_c} & 1 \end{pmatrix} \begin{pmatrix} 0 & -f_c \\ \frac{1}{f_c} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1+f_c C}{f_c D} & 1 \end{pmatrix}$$

The first of these is shorter but clearly fails if  $B = 0$ . Since the determinant is equal to 1 then  $B$  and  $D$  cannot both be zero. The value of  $B$  is the test that determines which algorithm is used. We use either 5 steps or 6 as the case may be.

Chirp Transform maker

Fresnel
Frac. Fourier
A B C D

Linear canonical transform

$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$  The ray matrix should have determinant 1.

Oversize Fourier Space

Use batch file

A

B  (mm)

C  (1/mm)

D

$\lambda$   ( $\mu\text{m}$ )

pixel sep.  (mm)

Hide window

Case  $B \neq 0$ .

- 1) lens function
- 2) inverse FFT
- 3) lens function
- 4) FFT
- 5) lens function

$$\phi_1 = \frac{\pi \Delta^2 (A - 1)}{\lambda B} (n^2 + m^2)$$

$$\phi_3 = \frac{-\pi B \lambda}{N^2 \Delta^2} (n^2 + m^2)$$

$$\phi_5 = \frac{\pi \Delta^2 (D - 1)}{\lambda B} (n^2 + m^2)$$

$\Delta$  is the pixel separation &  $f_c = N \Delta^2 / \lambda$ .

No attempt is made to assure that the determinant of the matrix is actually equal to one. You can cheat all you want as long as you get the right answer.