HOW TO PUT AMPLITUDE INFORMATION ON A PHASE ONLY DEVICE

It is very difficult to align two light valves so that one will modulate amplitude and the other phase so as to produce a fully formed hologram. If one could and the light valves were perfectly efficient it would be possible to encode the fresnel transform of any image upon the combination. So we compromise. We choose to just display a phase-only object and try to encode the phase in such a way that some of the image resulting from the phase representation looks like what we would like to see.

Suppose we start with an already rasterized image consisting of an array of complex numbers. This is most likely the Fourier transform of some image.

$$F_{nm} = R_{nm} e^{i\phi_{nm}}. \tag{1}$$

Where n and m are the integer coordinates of the pixels. This is not the phase-only object that we can present to our light valve. So we need some tricks. There are basically two approaches that we have had some success with and which I will outline bellow. Both can be extended under special circumstances.

1. PHASE ENCODING THROUGH THE SINC FUNCTION

So as a starting point assume that we have a phase only function of the form below.

$$e^{iS_{nm}\phi_{nm}} \tag{2}$$

The phase ϕ_{nm} is assumed to be in the range $-\pi < \phi_{nm} < \pi$ and $0 \le S_{nm} < 1$.

At every point on the array we find a Fourier series

$$e^{iS_{nm}\phi_{nm}} = \sum_{k=-\infty}^{\infty} a_{k,nm} e^{ik\phi_{nm}} \tag{3}$$

Treating the ϕ_{nm} as a variable we solve for $a_{k,nm}$.

$$a_{k,nm} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(S_{nm}-k)\phi_{nm}}\ d\phi_{nm} = \frac{\sin(\pi(S_{nm}-k))}{\pi(S_{nm}-k)}$$

And then...

$$e^{iS_{nm}\phi_{nm}} = \sum_{k=0}^{\infty} \frac{\sin(\pi(S_{nm}-k))}{\pi(S_{nm}-k)} e^{ik\phi_{nm}} \tag{4}$$

So, what have we got here? The whole expression is phase only but the summation is intimidating. But it is really very cool. Here's why. if we focus our attention on the $k = 1$

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