$$LG_n^l(r,\phi,z)=\sqrt{\frac{2\ n!}{\pi(|l|+n)!}}\frac{(\sqrt{2}r)^{|l|}}{(\omega(z))^{|l|+1}}\exp\left(\frac{-r^2}{\omega^2(z)}\right)L_n^{|l|}\left(\frac{2r^2}{\omega^2(z)}\right)$$

$$\times\exp\left(\frac{ikr^2}{2R(z)}\right)\exp(il\phi)\exp\left(-i(2n+|l|+1)\zeta(z)\right)$$

$$\mathrm{w}here\ \omega(z)=\sigma\sqrt{1+\left(\frac{z\lambda}{\pi\sigma^2}\right)^2}\ ,\ R(z)=z\left(1+\left(\frac{\pi\sigma^2}{z\lambda}\right)^2\right)$$

$$,and\ \ \zeta(z)=\arctan\left(\frac{z\lambda}{\pi\sigma^2}\right)\ ,\ \sigma=\ \mathrm{b}eam\ waist$$